EXAMPLES OF GEOELECTRIC SURVEYS

By Professor L. S. PALMER, D.Sc., Ph.D., F.Inst.P., Member.

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SUMMARY

The Introduction deals briefly with the general principles underlying geoelectric methods of surveying and gives a simple formula for the apparent resistivity at a given depth below the surface in a homogeneous medium. The two problems of interpreting the different kinds of curves obtained from field measurements and of determining the depth of the associated geological anomalies are discussed.

In Section 1.2 the electrical equipment and field procedure are briefly described.

Three particular anomalies are considered in Section 1.3. These are: (i) Parallel strata as illustrated by the boulder-clays which overlie the chalk of the Holderness Plain, East Yorkshire.

(ii) Inclined strata which occur where the water-table reaches the surface on the slopes of the Mendip Hills, Somerset.

(iii) A cavern in the limestone rocks at Pen Park near Filton, Bristol. In Sections 2, 3 and 4 respectively, the relevant equations, the geoelectric measurements and graphs together with their interpretations are discussed. The equation for inclined strata is deduced in a form comparable with the previously published equations for parallel strata and a spherical cavity. These latter are stated in Sections 2 and 4 respectively, and their derivation is given in the Appendix.

From the graphs in Section 2 it is concluded that no ancient Humber estuary existed in the chalk beneath the boulder-clays, but the presence of an interglacial gravel separating two deposits of boulder-clays is established.

In Section 3 the water-table is located and is found to emerge along a line of springs which coincides with the edge of the conglomerate deposited by the ancient Triassic sea.

The deductions made from the graphs of Section 4 were verified by subsequent excavations which led to the rediscovery of Pen Park Hole.

In the Appendix the derivation of the equations for a homogeneous medium, for parallel strata and for a buried sphere are given, together with theoretical relationships for calculating the depths of these particular anomalies.

LIST OF SYMBOLS

- a = Half the distance between the current electrodes, ft.
- $a_0 = A$ critical value of a.
- b = Half the distance between the potential electrodes, ft.
- d = Distance, ft.
- h = Depth below surface or boundary, ft.
- I = Current, amp.
- $k = (\rho_2 \rho_1)/(\rho_2 + \rho_1) =$ Reflection coefficient of the boundary between two media 1 and 2.
- n = An integer.
- n_e = Number of effective images formed by inclined mirrors.
- r =Radius of a circle of images or of a spherical cavity, ft.
- R = Measured resistance, ohms.
- V_P = Potential at a point P, volts.
- $\alpha = b/a < 1.$

 $\beta = h/a = (r \sin \theta)/a.$

- $\rho_0 = \text{Resistivity of air} = \infty$.
- $\rho_s =$ Apparent resistivity at a depth *h*, ohm-ft.
- $\rho_x = \text{Resistivity of medium } x$, ohm-ft.
- $\hat{\theta} =$ Angle between inclined strata.

(1) INTRODUCTION

(1.1) General Principles

It is over forty years ago that Wenner¹ first published his particular technique for geoelectric surveying. His method is occasionally employed to-day² where the circumstances are thought to be suitable. Since that time (1916) many modifications have been suggested but the general underlying principles are the same. Briefly, an electric current is sent into the earth through two electrodes C_1 and C_2 inserted in the surface to a depth of a few inches. The earth with the two current electrodes constitutes a form of three-dimensional potentiometer over the surface of which is a potential 'pattern'. This pattern depends on the current distribution, which in turn depends on the variations of the electrical resistivity throughout the half-space bounded by the earth's surface and extending downwards to infinity, the upper half-space being air of infinite resistivity.

The potential fall between any two points on the surface can be measured by the insertion of two additional probes or potential electrodes P_1 and P_2 at the points between which the potential fall is to be measured. The ratio of the measured potential difference to the current flowing is a function of some 'mean' and localized value of the earth's electrical resistivity.

This 'mean' value will depend on the variations of the electrical properties of those portions of the earth's crust through which the major part of the current passes; whilst the electrode configuration on the surface will determine the location of the particular region, the mean resistivity of which can be calculated from the potential/current ratio, or 'potential function' as it is usually termed.

The name 'apparent resistivity' is used to distinguish this 'mean' value from the normal resistivity that would be determined by laboratory measurements on different samples of the rocks comprising the earth's crust.

Of the many suggested electrode configurations, some involve variable electrode spacings with or without one or more electrodes fixed in position throughout a set of measurements. Often when the field technique is simple and quick to use, the computations are the more tedious and complicated. Whatever method is used, the objective is to obtain a set of apparent resistivity measurements at different depths below the point on the surface (the station) in the neighbourhood of which the electrodes are situated.

With the original Wenner method, the four electrodes were equally spaced along a straight line bisected by the station, so that $C_1P_1 = P_1P_2 = P_2C_2 = d$. The apparent resistivity at a region immediately below the station is then given by

$$\rho_s = 2\pi dR \text{ ohm-ft} \quad . \quad . \quad . \quad . \quad (1)$$

where d is measured in feet and R is the ratio of the measured potential fall in volts to the circulating current in amperes.

It will be obvious that the depth below the station of the region of apparent resistivity ρ_s will increase with increase in the spread of the current electrodes. This follows because the current will penetrate more effectively to a greater depth when the distance C_1C_2 is large. It has been customary with the

Prof. Palmer is Professor Emeritus in the University of Hull and Honorary Curator, Wells Museum.

Wenner technique to assume that the depth of this region to which ρ_s refers is equal approximately to half the distance C_1C_2 .

The method used throughout the surveys described below is a generalized form of the Wenner method, in which the four electrodes are aligned symmetrically about the station, but, instead of being equidistant, the ratio P_1P_2/C_1C_2 is kept constant throughout any given series of observations. The apparent resistivity below the station is then given by [see Appendix 7.1]

$$\rho_s = \frac{\pi (1 - \alpha^2)}{2\alpha} aR \text{ ohm-ft} \quad . \quad . \quad (2)$$

where α is the ratio P_1P_2/C_1C_2 and is less than unity, *a* is $C_1C_2/2$ ft and *R* is the ratio in ohms of input voltage to output current.

If α is made equal to $\frac{1}{3}$, eqn. (2) reduces to eqn. (1) for the Wenner technique. The ability to choose different values of α for different sets of readings makes the present method very flexible. This is useful in the field because the larger the value of α , the greater is the measured potential. Consequently the same instrument can be used with equal accuracy on low-resistance wet peat or on high-resistance dry limestone. Furthermore, with large values of α , Huber³ has shown that variations of measured resistivities due to surface irregularities are reduced to a minimum.

It is convenient to plot values of a as ordinates against values of ρ_s as abscissae calculated from eqn. (2). By plotting in the second quadrant (i.e. a minus), the ordinates read downwards as functions of the depths and the abscissae increase to the right with the usual system of Cartesian co-ordinates.

Thus, when the station is situated above a homogeneous medium, i.e. one with uniform resistivity, the resulting graph will be a straight line parallel to the ordinate axis and intercepting the axis of abscissae at a value of ρ_s depending on the electrical nature of the rocks beneath the station. The equation of this graph is $\rho_s =$ (a constant) for all depths.

Variations or anomalies in the resistivity of the ground below the station, such as are produced by changes in geological structure, or by the presence of water, metal ore deposits, air cavities, etc., will produce more or less complicated curves from which the type of anomaly and its depth are to be determined.

Thus the two major theoretical problems in geoelectric surveying are:

(i) To associate particular types of curves with given geological anomalies.

(ii) To relate the ordinate values of the points of inflection or changes in slope with the actual depth below the surface at which the associated geological anomaly occurs.

The first problem can be studied from families of curves of the appropriate potential function, and the second problem entails the solutions of the second and first differentials respectively of the appropriate potential function.

For many years empirical rules were employed, based largely upon experience in the field. Examples of such rules are the Gish-Rooney law and the Peak rule.⁴

With the three particular anomalies dealt with in the following Sections, attempts have been made (see Appendix) to determine the theoretical relationships between the ordinate values of changes in the graphs and the depths of the associated anomalies below the surface of the earth. In many cases it has been possible to compare conclusions arising from these relationships with bore-hole data or with measurements based upon subsequent excavations.

(1.2) Apparatus

Because of serious interference from earth currents and contact potentials, a simple battery-operated circuit connected across the current electrodes and a voltmeter connected across the potential electrodes will not give reliable results. Furthermore, it would be convenient to make observations directly in ohms. Obviously from eqn. (2), a voltmeter could be calibrated in ohms if the current were kept constant. This is possible with suitable current controls. Methods must also be devised to inhibit the effects of earth currents and contact potentials where the electrodes are inserted in the ground.

The type of circuit finally adopted for the work described in this paper is shown in Fig. 1. A manually-operated direct-



Fig. 1.—Circuit diagram of electrical equipment.

current generator passes a current from winding N1 to the earth through the current electrodes C_1 and C_2 . The current is registered by the ammeter A and kept constant by adjusting the resistor R. S_1 and S_2 are mechanically-operated low-frequency reversing switches. The resulting low-frequency potential generated between P_1 and P_2 is mechanically commutated by switches S_3 and S_4 , with the result that a direct current passes through the galvanometer G₁. This circuit is protected by condensers as shown, from direct earth currents and from potentials set up at the electrode-earth contacts. Another direct current from winding N_2 of the generator passes through the potentiometer P in such a direction that it is opposed by the direct current coming from the potential electrodes. By adjusting the slider of the potentiometer, the galvanometer G_1 can be used as a null-recording instrument. The back-current from the generator necessary to do this is passed through the deflecting coil D of the galvanometer G_2 , the controlling field of which is provided by the constant current I in the coil C. Thus, the deflection of the needle will be proportional to the current through D, i.e. to the potential generated across P_1P_2 . Since I is kept constant, this deflection can be calibrated directly in ohms, and the galvanometer G₂ thus functions as an ohmmeter.

The adoption of low-frequency reversing and commutating switches, which are controlled by the revolutions of the manuallyoperated handle of the generator, overcomes the troubles due to earth currents and contact potentials without introducing the reactance and phase difficulties associated with a.c. bridge methods.

For simplicity, the various shunt and series resistances introduced to extend the ranges of the instruments have been omitted from the diagram.

Concerning the electrodes, for most top-soils $\frac{1}{2}$ in diameter steel rods are suitable. One end is pointed and preferably casehardened, and the upper end is looped round to form a convenient handle. The overall length can be about a foot. Such electrodes can be pushed into the ground by hand. It is convenient to connect the cable by inserting the end of the wire into a hole drilled through the electrode, the wire being secured by a grub-screw.

For hard rock it is necessary to drive electrodes in with a small sledge hammer. Suitable electrodes for this purpose can be made from $1\frac{1}{2}$ in diameter steel tubes, the lower ends of which are plugged and welded with case-hardened steel points. A steel driving bar, which fits loosely into the tubular electrode and protrudes several inches above it, is used for driving the electrode into the rock. With this design, the top of the electrode is undamaged whilst the burring over of the top of the driving bar does not matter. The cables are conveniently fastened to small iron collars which slip over the electrodes, tommy-bars can be inserted through holes drilled at the top of the tubes. The overall length of the electrodes need not exceed 15 in.

Four heavy t.r.s. cables wound on suitable drums, and measuring tapes, complete the essential field equipment.

With the technique adopted in the work here described, all four electrodes need to be moved for each measurement. For this operation it is convenient to employ four people. A minimum of two others is necessary to operate the station: one adjusts the potentiometer and records the ohmmeter readings, and the other operates the generator and signals to those moving the electrodes.

To facilitate the calculation of ρ_s from eqn. (2), a slide-rule has been designed from which this quantity can be read directly with only one adjustment of the slide. This has been of considerable value in the field when graphs need to be plotted and interpreted before more stations can be sited. The design of the slide-rule depends on the fact that α in eqn. (2) is kept constant for a given series of readings. Eqn. (2) can be written in the form

$$\frac{\rho_s}{R} = \frac{a}{f(\alpha)}$$
 where $f(\alpha) = \frac{2\alpha}{\pi(1-\alpha^2)}$

It is now apparent that an ordinary slide-rule with the fixed scale appropriately marked with values of $f(\alpha)$ can be used to read off values of ρ_s against recorded values of R. The necessary adjustment is to move the slide so that a given value of a coincides with the particular value of $f(\alpha)$ in use. The value of ρ_s on the slide then coincides with the particular value of R on the fixed scale, a and R being corresponding field readings.

(1.3) Anomalies

The three anomalies discussed below are:

(i) Parallel strata of different thicknesses and resistivities: in particular the boulder-clays of the Holderness Plain, East Yorkshire, which overlie the chalk and are overlain by top-soil. A thin gravel seam usually separates the two different boulder-clays, which were deposited during successive glacial climatic phases.

(ii) A water-table marked by a line of springs rising on the side of a steep hill: in particular the water-table at Link Batch near Burrington Combe on the Mendip Hills, North Somerset, where the upper dry limestone overlies a wet limestone and results in a series of springs debouching along the north side of the hill.

(iii) A cavern containing water: in particular the cavern known as Pen Park Hole near Filton, Bristol. The cavern is situated in limestone which is covered by a layer of clay. A lake lies in the bottom of the main chamber, which has several passages and tunnels connected to it.

(2) HOLDERNESS BOULDER-CLAYS

Geoelectric measurements were made across the Holderness Plain, East Yorkshire, during the post-war years and were completed in 1954.

(2.1) Objectives

It has been suggested that the River Humber once flowed into the North Sea by way of a wide estuary situated south of Skipsea some 20 miles north of the present river mouth. To test this hypothesis was one of the objectives of the survey. It was proposed to measure the depth of the low-resistance boulder-clays and thereby to plot the contours of the underlying relatively high-resistance chalk.

Additional objectives were to determine the position and continuity of the interglacial gravels which were believed (from bore-hole observations) to separate the upper and lower boulderclays. It was also thought that the distribution and depth of the boulder-clays would yield information concerning their deposition and also lead to further knowledge about the lateral and terminal moraines with which they are associated.

(2.2) Theory

The theory of resistivity measurements in localities below which lie parallel strata of different thicknesses and resistivities has been published by many authorities.⁵⁻⁸ The general formula for the apparent resistivity ρ_s at different depths in a two-layer system is (using the above nonenclature)

$$\rho_{s} = \rho_{1} \left[1 + \frac{(1 - \alpha^{2})}{\alpha} \sum_{n=1}^{n=\infty} k^{n} \left\{ [1 - \alpha)^{2} + (2n\beta)^{2} \right]^{-1/2} - [(1 + \alpha)^{2} + (2n\beta)^{2}]^{-1/2} \right]$$
(3)

where ρ_1 is the resistivity of the top layer of thickness h, ρ_2 is the resistivity of the lower layer of infinite thickness, β is the ratio h/a and $k = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is usually known as the reflection coefficient from its obvious analogy with the optical case (see Appendix 7.2).

Typical graphs for two and three layers are given by most authors, and the particular relationship between points of inflection on the graphs and the depths of the boundaries between adjacent layers has been determined by Palmer and Hough (Reference 4, Fig. 2). With this knowledge it is possible to calculate the depths of the several boundaries in a multi-layer system.

(2.3) Technique and Survey

The depth to which measurements can be taken depends on the extreme distance between the current electrodes and corresponds very roughly to half this distance. As the current electrodes are brought nearer and nearer to the station, so the potential electrodes are moved to such positions that α is kept constant and always less than unity. For low-resistance boulder-clays it was convenient, with the instruments available, to make α equal to 0.7 or 0.8.

Readings were usually taken for values of a from 5 to 380 ft, at which latter distance the effects of the chalk on the shape of the graphs were fully apparent. Graphs were then plotted with the values of ρ_s calculated from eqn. (2) as abscissae and with the distances a as ordinates.

Fig. 2 shows four lines running approximately north-south along which 44 stations were sited. At most of these stations two traverses were laid out, one parallel to the line of stations and one perpendicular to it. The use of two mutually perpendicular traverses served as a check on the measurements and confirmed any minor anomalies arising from the occurrence of any unsuspected stratum (such as a layer of low-resistance peat or a seam of high-resistance sand or gravel) in what was assumed to be a uniform deposit of boulder-clay of constant resistivity.

(2.4) Results

Apart from local variations depending on the depth of the chalk, the graphs for 30 of the stations (for example, B13) are three-layer types that might be expected from the known geological structure of the Holderness Plain. In most of these three-layer graphs the minimum value of ρ_s is not less than 100 ohm-ft.

Of the remaining 14 stations, a group of 7 stations (enclosed in circles in Fig. 2) lie on a roughly semicircular or horse-shoe-



Fig. 2.—Distribution of stations on the Holderness Plain.

shaped line running from Ulrome westwards through Beeford and North Fordingham, thence southwards towards Burshill and possibly as far south as Routh before turning back eastwards through Sigglesthorne towards Hornsea. The graphs for these stations (for example, B10) conform more closely with two-layer graphs than with the three-layer types. Their straight portion conforms roughly with the equation $\rho_s \simeq 100$.

The remaining 7 stations (enclosed in squares in Fig. 2) are all situated immediately north and east of Hull and in Hull itself. The graphs (for example, B2) are characterized by very low minimum values of ρ_{s} , usually less than 30 ohm-ft.

It is not possible in this paper to reproduce the graphs for the 44 stations. The graphs for three stations on the line BB', however, are shown in Fig. 3. These have been selected as typical of the three types mentioned above. Furthermore, in the locality of each of these stations was a bore-hole, the geological section of which could be compared with deductions made from the respective graphs. These sections, which are shown in Fig. 3, have been compiled from the Geological Survey Memoir on 'Holderness' and from Wartime Pamphlet No. 12 on 'The Water Supply from Underground Sources in East Riding and North Lincolnshire', by K. P. Oakley and others. By selecting stations all on the same line (BB') it is possible to see how the interpretation of their graphs has contributed points on the N-S section along BB' which is drawn in Fig. 4.

(2.5) Interpretations

The geological significance of this geoelectric survey will now







Fig. 4.-S-N Section along BB' from geoelectric measurements.

be indicated. Comparisons with the bore-hole data will then show to what extent a survey of this kind can be of practical use to engineers and geologists.

In order to calculate the depth of the chalk or the thickness of the total overburden it is necessary to determine for the appropriate value of k the value of $\beta(=h/a)$ for the particular ratio of $\alpha(=b/a)$ in use. Then the product βa_0 (where a_0 is the value of a for the lower point of inflection of a three-layer graph) gives the required depth h. Details of this procedure are given in Reference 4.

There are two general conclusions which follow when the 30 three-layer graphs are considered as a whole. The first concerns the depth of the boulder-clay. Along the line BB' these glacial clays are roughly 100 ft deep, along CC' they are about 40 ft and they thin out to 20 ft along the line DD'. They also become very shallow as they approach the chalk wolds towards the north. There does not seem to be any evidence for a large pre-glacial estuary in the chalk as was once surmised.

The second general conclusion concerns the small highresistance kinks which appear in many of the graphs. They occur for values of a equal to about 60 ft along the line BB', and 20–30 ft on line CC', but could not be distinguished on the graphs for any of the stations along DD'. The interpretation of these kinks is assisted by a knowledge of the bore-hole and coastal sections. Frequently at depths corresponding to these values of a, strata of sand or thin gravel seams occur, and are considered to be evidence of an interglacial phase which marked the interval of time between the deposition of the lower and upper boulder-clays.^{9, 10} Thus these kinks appear to be geoelectric evidence for this particular interglacial period.

The two-layer graphs, such as B10 in Fig. 3, indicate that

deposits of very low resistivity occur below these stations. With some knowledge of the local geology it is reasonable to suppose that these low-resistance deposits arise from either abnormally wet clays or from soft wet broken chalk. Soft chalk has been recorded at several of these stations, and it is thought that the graphs are due to wet chalk rather than to wet clay. The so-called 'Beeford Clays' may be an exception to this. The distribution of these particular stations (see Fig. 2) coincides with the horse-shoe-shaped peripheral edges of a tongue of boulder-clay which is surrounded with terminal and lateral morainic gravels.¹¹

That these two-layer graphs appear to extend to indefinite depths is due to the fact that any wet low-resistance deposit acts as an electric screen effectively preventing the penetration of the current to lower formations. This is particularly noticeable with subterranean lakes in limestone caves, where the graphs appear to suggest an indefinite depth of water, even though the lake might be very shallow (see Section 4). Water-logged deposits are consequently effective barriers against deeper geoelectric measurements. Thus these two-layer graphs indicate the position of wet broken chalk, and, together with the superficial morainic gravels, mark the western advance of the particular tongue of boulder-clay with which they are associated.

The geological explanation of the seven remaining graphs lies in the fact that all the stations are situated on the very lowresistance alluvial deposits which mark the older courses of the Hull-Humber river system.

These examples are illustrative of the simple two- and threelayer types of graphs which arise from a number of superimposed parallel strata. Graphs somewhat similar to the above have recently been obtained by Habberjam and Whetton on the boulder-clays in Co. Durham.¹²

(3) MENDIP WATER-TABLE

The measurements described in this Section were made during the summer of 1948 in order to test the theory which had been developed in the spring of that year.

(3.1) General Problem

This work was undertaken as an example of the application of geoelectric methods to the problem of inclined strata. It was known that the resistivities of wet and dry limestone were sufficiently different for the boundary water-table to be treated as the boundary between two geologically different strata. Consequently, where a water-table comes to the surface—a location usually marked by a line of springs—there will be an intersection between two inclined boundary planes. This situation is illustrated in the diagrammatic section depicted in Fig. 5.



Fig. 5.—Diagrammatic section through Link Batch, Mendip Hills showing the four stations.

A water-table is not necessarily horizontal.* It often rises towards the surface, and, with the site chosen, the surface itself was also inclined to the horizontal. We thus have a situation appropriate for testing the theory underlying the problem of inclined strata.

(3.2) The Site

The foregoing conditions arise on the north and south sides of the Mendip Hills, where, towards the foot of the hills, there is a line of springs. These springs occur where impervious Triassic conglomerate rocks overlie the mountain limestone. Below the line of springs the limestone is wet, and above that line it is relatively dry.

A site was chosen on the north face of the Mendips near the bottom of Burrington Combe. Here, at Link Batch, open fields enable traverses to be sited without obstruction by vegetation. Four traverses were selected, as shown in Figs. 5 and 6, and



Fig. 6.—Plan of Link Batch traverses.

were chosen so that stations 1 and 2 were above the dry/wet limestone boundary and the lower stations 3 and 4 were on the conglomerate.

(3.3) Theory

It is proposed to give in this Section a brief outline of the theory concerned, based on the method of electric images. This was first attempted by R. F. Aldredge.¹³ Later Miss Skalskaya,¹⁴ M. Unz,¹⁵ Katsuro Maeda¹⁶ and J. C. de Gery, with G. Kunetz,¹⁷ derived more rigorous solutions involving Bessel and similar functions.

The theory of inclined strata involves the problem of the effective number of multiple images produced between two plane mirrors inclined at any angle. Consider station 2 in Fig. 5. The simplified equivalent problem using Maxwell's theory of electric images is indicated in Fig. 7, in which, for convenience, the hillside is shown horizontal. The circles are the loci of the electric images arising from the equivalent current source and sink of +2I and -2I at C_1 and C_2 respectively. P_1 and P_2 are positions of the potential electrodes. ρ_0 , ρ_1 and ρ_2 are the resistivities of air, dry limestone and wet limestone respectively, ρ_1 being greater than ρ_2 . Because of these resistivity values, the two 'mirrors' will have different reflection coefficients; that of the dry-limestone/air boundary is $(\rho_0 - \rho_1)/(\rho_0 + \rho_1)$ and is equal to +1 because ρ_0 is equal to infinity. This mirror is therefore opaque, with a back coefficient of -1. The reflection coefficient of the water-table will be -k and equal to $(\rho_2 - \rho_1)/(\rho_2 + \rho_1)$. This mirror is therefore semi-transparent

^{*} When water falls on a hill it may sink as vadose water to a level appreciably below that of the phreatic water which emerges along the sides of the hill. The surface of the water-table is consequently not necessarily a horizontal plane, but is often saucer-like. The curved surface of water confined within the solid rock should not be confused with the horizontal surface of the free water in an open pond.



Fig. 7.—Diagram of inclined strata with traverse C_1C_2 and associated electric image circles.

and will consequently have a reflection coefficient of +k for 'light' passing in the reverse direction.

Referring now to Fig. 8, which shows some of the images due to the current +I at C₁, the centre of the circle corresponds to the line of intersection of the two mirrors and coincides with the line of springs in Fig. 5. As a result of multiple reflections, a series of images is formed round the circle, those connected by full lines being formed by reflections at the semi-transparent mirror of reflection coefficient -k and at the opaque mirror for which k = +1. Those images connected by broken lines are



Fig. 8.—Diagram illustrating the formation of electric images by multiple reflections between inclined plane reflecting boundaries.

formed by light passing in the reverse direction, when the coefficients of reflection are +k and -1 respectively. Fig. 8 depicts the special practical case when the source I is coincident with the ground/air boundary, which has a reflection coefficient of unity. As a consequence, the effective intensity of the source becomes 2I and the total number of possible images is halved. This follows because there is no virtual image of I above the ground surface to act as a separate source with its own independent series of images.

The following conditions then pertain:

(i) Each pair of images is of strength $2k^{n}I$.

(ii) Each pair of images is of situated $2r \sin n\theta$ from the source, where r is the distance of the station from the line of intersection of the reflecting boundaries and θ is the angle between them.

(iii) Each pair of images and the source subtend an angle of $2n\theta$ at the centre of the circle.

(iv) Many pairs of images are neutralized (as, for example, $\pm 2k^3I$ in Fig. 8). When higher orders are not neutralized, they

can usually be neglected because k is less than unity and consequently the image intensities soon become negligibly small. (v) The total number of images depends on the ratio of π to θ .

(v) The total number of images depends on the ratio of π to θ . But whereas the number of images must necessarily be a whole number, the ratio π/θ will, in general, be fractional.

In practice, the effective number of images n_e is given by the largest integer less than π/θ . This number includes the first image to occur in the back angle, and it determines the upper limit of integration in eqn. (4).*

It is now possible to calculate approximately the potential at any point P due to the series of electric images arising from the current I passing between the electrodes C_1 and C_2 , and thence to determine the potential difference V between any two particular points, such as P_1 and P_2 :

From Fig. 7 it follows that

$$V_{P1} = \left[\frac{\rho_1(2I)}{4\pi(a-b)} + \frac{\rho_1(2I)}{4\pi} \sum_{n=1}^{n=n_e} k^n / d_1\right] \\ + \left[\frac{\rho_1(-2I)}{4\pi(a+b)} + \frac{\rho_1(-2I)}{4\pi} \sum_{n=1}^{n=n_e} k^n / d_2\right]$$

where $d_1 = [(a - b)^2 + (2r \sin n\theta)^2]^{1/2}$

and
$$d_2 = [(a + b)^2 + (2r \sin n\theta)^2]^{1/2}$$

Therefore
$$V_{P1} - V_{P2} =$$

$$\frac{2bI\rho_1}{\pi(a^2-b^2)} \left[1 + \frac{a^2-b^2}{b} \sum_{n=1}^{n=n/2} k^n \left\{ [(a-b)^2 + (2r\sin n\theta)^2]^{-1/2} - [(a+b)^2 + (2r\sin n\theta)^2]^{-1/2} \right\} \right]$$

By using eqn. (2) and putting $b/a = \alpha$ and $(r \sin \theta)/a = h/a = \beta$ this reduces to

$$\frac{(V_{P1} - V_{P2})}{I} \frac{\pi (1 - \alpha^2)}{2\alpha} a = \rho_s =$$

$$\rho_1 \left[1 + \frac{1 - \alpha^2}{\alpha} \sum_{n=1}^{n=n_s/2} k^n \left\{ \left[(1 - \alpha)^2 + \left(2\frac{\sin n\theta}{\sin \theta}\beta\right)^2 \right]^{-1/2} - \left[(1 + \alpha)^2 + \left(2\frac{\sin n\theta}{\sin \theta}\beta\right)^2 \right]^{-1/2} \right\} \right] \quad (4)$$

When θ becomes zero, the summation limit goes to infinity and eqn. (4) reduces to eqn. (3) for parallel strata. Eqn. (4) is comparable with Unz's¹⁵ eqn. (16) derived for the Wenner method using equally-spaced electrodes.

The complicated relationships between the points of inflection and peak values of the graphs of eqn. (4) and the actual values of k, θ and h have been determined from the solutions of the equation $d^2\rho_s/da^2 = 0$. These solutions were obtained by methods similar to those used for parallel strata.⁴

(3.4) Results

The graphs obtained at the four stations shown in Figs. 5 and 6 are reproduced in Fig. 9. It is at once apparent that the graphs for the two upper stations are characterized by a highresistance layer ρ_1 —the dry limestone—which overlies a lowerresistance one ρ_2 —the wet limestone. The reflection coefficient $(\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is therefore negative. This is contrasted with the graphs for the two lower stations, where k is positive and a relatively low-resistance deposit—the conglomerate—overlies the same wet limestone.

The resistivity of the top-soil (the abscissa value when

 \bullet If more images are included, values of ρ_s become negative and the graphs pass into the third quadrant.



Fig. 9.—Graphs of the four stations on Link Batch.

 $a \rightarrow$ zero) shows a progressively lower value as the altitude of the station gets less.

The values of ρ_s for the larger values of *a* are not, as in the case of two parallel strata, simply related to the resistivity ρ_2 of the lower stratum. They are fractional values of the resistivity ρ_1 of the upper deposit for negative values of *k* and multiple values of ρ_1 when *k* is positive. Furthermore, they are also functions of the distance *r* of the station from the line of springs, i.e. from the apex of the angle θ .

Another point of interest arises when it is realized that the difference in the depths of the water-table below stations 1 and 2, namely 14 ft (α and k both being equal to about 0.5), exceeds their difference in altitude, namely 10 ft, the thicker layer of dry limestone being beneath the upper station. Consequently the water-table slopes upwards to the line of springs which rise to the surface between stations 2 and 3 (see footnote to Section 3.1).

In a similar way, the thickness of the conglomerate decreases in the uphill direction and its boundary tends to intersect the hillside approximately along the same line of springs. These conclusions are drawn to scale in Fig. 10.



Fig. 10.-S-N section on Link Batch from geoelectric measurements.

(4) PEN PARK HOLE

In August, 1956, a geoelectric survey was undertaken at the request of the City Engineer at Bristol. It was thought probable that a large cavern was situated somewhere beneath certain fields upon which it was proposed to build houses. There was no evidence on the surface to suggest where such a cavern might be, but early records (18th century) referred to a deep rift which opened out into a large cave. The rift had been closed after the death in 1775 of a Canon of Bristol Cathedral who fell down the rift into a lake at the bottom of the cavern.

The purpose of the survey was to locate the cavern, and, if possible, to determine the thickness of the limestone at the point where the roof of the cave came closest to the surface.

(4.1) Historical

The cavern in question is known as Pen Park Hole; a description, with a survey undertaken by a Mr. White in 1779, was written by Mr. A. S. Catcott and published by Rudhall of Bristol in 1792. This survey indicates the main entrance to the cavern—the rift mentioned above—and two other entrances. One entrance was down a shaft into a miners' artificial tunnel. This was known as the Eastern Entrance and was probably made and used by medieval lead miners. The other entrance was west of the main rift. All three entrances had disappeared leaving no trace on the surface, but gruffy ground indicated the probable line of a long rift which extended approximately E–W across the countryside.

With the help of White's original survey (preserved in the Bristol Reference Library) the position of the first station was decided upon. This site subsequently proved to be only about 50 ft from the highest point of the main chamber.

(4.2) Local Geology

The interpretation of graphs of geolectric data needs to be guided by a knowledge of the local geological structure. For example, the presence of a fault will produce a graphical variation which will be superimposed upon the variation produced by any other anomaly. This superposition of two anomalies may result in a distortion or in a change of magnitude of the effects produced by either anomaly separately. The interpretation of complex variations is often extremely difficult unless one of them can be recognized and its effect appreciated.

In a similar way, surface effects produced by cliffs, quarries or railway cuttings may be so great as to mask the smaller effect of the particular anomaly being studied. See, for example, the top part of the graph for station 1 in Figs. 6 and 9.

For this reason it is advisable to obtain at some nearby site a graph characteristic of the locality, and then survey the anomaly, such as a cave, salt dome, lead vein, etc. The second graph can be compared with the standard or characteristic graph of the immediate neighbourhood, and the form of the graphical kink due to the anomaly can be appreciated and properly interpreted.

In the neighbourhood of Pen Park Hole the limestone protrudes as an anticline in a surrounding 'sea' of Liassic clay which overlies it at its lower reaches. In many of the traverses, one current electrode may be above limestone whilst the other is in the Lias. This will cause a distortion in the lines of current flow, the current naturally concentrating on the path of least resistance. Similarly, the current would tend to flow in any low-resistance path when such a path was presented to it. Cases of this kind need considerable caution and much experience to interpret their graphs. A good example arose when an attempt was made to trace a tunnel under Pen Park Road. One traverse happened to run almost along a buried water main. This presented a very low-resistance path along which the current flowed, with the result that the effect of the underlying tunnel was completely masked.

In the neighbourhood of Pen Park the Liassic clay becomes thicker and thicker the farther it is from the limestone outcrop. If caves occurred in the limestone underneath the clay, they would be a long way below the surface and masked by the very low resistance of the clay overburden. Consequently it is unlikely that they could be detected by a geoelectric method. Reference was made to this geological situation when considering the results in Section 2.5.

(4.3) Theory

J. N. Hummel in 1930 obtained an equation for the apparent resistivity of a spherical cavity in a homogeneous medium.⁵

But he treated the special case of fixed current electrodes spaced apart a distance equal to four times the depth of the cavity, and such a very special case is of little use since it presupposes a knowledge of the depth of the cavity. In 1953 Van Nostrand¹⁸ using bipolar co-ordinates considered the case of a buried conducting sphere and came to the conclusion that it is not feasible to detect a conducting body buried to a depth greater than the mean of its linear dimensions. This is not borne out in practice with high-resistance bodies.

A different treatment yielding an equation comparable with eqns. (3) and (4) was published in 1954.¹⁹ A sphere of radius r full of air of infinite resistivity was considered to be buried with its centre at a depth h below the horizontal boundary of a medium of resistivity ρ , the half-space above the boundary being air.

The result obtained using the comparable analogy of an electric doublet in an unbounded medium of the same resistivity was

$$\rho_s = \rho \left\{ 1 + \frac{\beta^3 (1 - \alpha^2) (r/h)^3}{\alpha^3 (1 + \beta^2)^{3/2} [1 + (\beta/\alpha)^2]^{3/2}} \right\} \quad .$$
 (5)

where $\alpha = b/a$ and $\beta = h/a$.

The form of the graph produced by this particular anomaly is



Fig. 11.—Theoretical graphs showing the effect of A. A high-resistance cavity in a homogeneous medium. B. A high resistance cavity in a two-layer system. C. As for B but with water in the bottom of the cavity.

shown in curve A, Fig. 11. By equating to zero the first differential with respect to a, the equation

is obtained, a_0 being the value of a for the maximum (or minimum) value of ρ_s . Thus, since α is less than unity, it is apparent that the anomalous bulge appears lower down on the graph than the depth of the cavity from the surface.

When the cavity is in, say, limestone covered with a thin layer of relatively low-resistance top-soil, the graph takes the form shown in curve B of Fig. 11. Curve C shows the effect of water in the bottom of the cavity. Although the water may be only a few inches deep, its resistivity (especially in limestone caves) is very low. Consequently, the sheet of water acts like an electric screen (a copper sheet, for example). Hence, although the presence of water can be detected quite easily, its depth cannot be measured by geoelectric methods. In other words, the graph does not bend back to the normal value for limestone, but continues indefinitely downwards at the low value of the resistivity of water. This effect was discussed when referring to wet broken chalk in Section 2.

(4.4) Survey

Work was commenced on the 7th August, 1956, and by the 15th August the main cavern had been located and a lake was

detected at the north-east end of the cave. The original Rift Entrance and the position of the shaft leading to the Eastern Passage were also located. The Eastern Passage, by a series of short traverses, was traced from the bottom of the shaft to where it entered the main chamber.

These conclusions were based upon 800 readings at 40 stations (Fig. 12) taken over a period of 7 days. Measurements were



Fig. 12.—Distribution of stations round Pen Park Hole. —|-|-| Clay/limestone boundary.

made to depths of 400 ft on the traverses over the main chamber, but only to 150 ft when tracing the course of the higher passage and when locating the original entrances. The Western Entrance was not located and may possibly have been covered by a new roadway or may be obscured by a nearby and recent hedgerow.

(4.5) Results

Even if it were desirable, it will not be possible to show all the graphs arising from this survey. Fig. 13 shows two typical graphs obtained from traverses above the main cavern. These are marked 5 and 6. The graph of station 5, which is comparable to curve C in Fig. 11, indicates the presence of water at the bottom of a large cave. At station 6 the lower portion of the graph tends to become asymptotic to $\rho_s \simeq 1500$, and consequently the water below station 5 does not extend below station 6. Station 5 was situated north of station 6. Subsequently it was found that the lake was confined to the north-eastern portion of the cave. For these graphs α was equal to 0.2; hence, using eqn. (6) and a value of a_0 of about 70 ft, the thickness of the cave roof in the neighbourhood of station 6 appears to be $\sqrt{(0.2)} \times 70$, or just over 30 ft. This has not yet been verified by internal measurements.

The three graphs 32, 27 and 19 in the top left-hand corner of Fig. 13 were taken over the Eastern Passage leading to the bottom of a filled-in mine shaft. The graph obtained almost immediately above the mine shaft is numbered 34. It will be noticed that the high-resistance tunnel or passage gets larger as it gets



Fig. 13.—Graphs for two stations above Pen Park Hole, for four stations above the shaft and Eastern Entrance and for one station on the Liassic clay.

deeper, while the shaft yields a typical vertical limestone graph up to $a \simeq 50$, above which the value of ρ steadily decreases to the low value of the in-filling. The depth of the shaft, with $\alpha = 0.4$ for these particular traverses, is therefore $\sqrt{(0.4)} \times 50$, or about 30 ft. The junction of the miners' tunnel with the shaft was subsequently found at this depth. Fig. 13(a) is a rough sketch showing the section as deduced from these four graphs.

Finally, station 38 was one of several sited over the Liassic clay at some distance from the limestone outcrop. The graph is interesting as being typical of an almost homogeneous low-resistance medium. A more detailed inspection indicates that the upper parts of the clay were probably wetter than the lower parts, whilst the top-soil has a resistivity comparable with that above the other stations. For these low-resistance localities it was convenient to make α equal to 0.8.

(4.6) Excavations

The above conclusions having been drawn by interpreting the graphs from the 40 stations, it was felt desirable to test them by actual excavations. Consequently it was decided to try to open up the Eastern Entrance because the work involved in attempting to open the main Rift Entrance would not only be too long a task but would probably be very dangerous. Excavation was commenced at Station 34, and the original miners' shaft was reached at a depth of about 5ft from the surface. The station had been sited almost exactly above the centre of the shaft. When the infilling of recent rubble was first removed, the passage predicted in Fig. 13(a) was missed by about 3 ft, and a lot of unnecessary digging was undertaken before a clay choke was located at a depth of 32 ft. When cleared, this led directly into the Eastern Passage, which followed the course already mapped by the geoelectric surface measurements. As suspected, the passage terminated on the side of the main chamber. The bottom of the cavern was reached at a depth of over 150 ft from the surface. A lake of muddy water was found at the north-east side below those stations which indicated the presence of water on their graphs. The exploration of the cavern and its several side-passages is continuing.

(5) ACKNOWLEDGMENTS

The extensive field work involved in the three examples of geoelectric surveys described in Sections 2, 3 and 4 could not

have been undertaken had it not been for the ready assistance I received from many colleagues, students and other friends interested in the work.

As will be appreciated, in order to carry out a geoelectric investigation by the technique adopted in these surveys, a team of at least six persons is desirable. It is therefore with much gratitude and pleasure that I am able to record my thanks to all those concerned, and in particular to the following:

Throughout the survey on the Holderness Plain, Mr. J. M. Hough, of the Physics Department at Hull University (then the University College), assisted with both the theoretical work and with the practical measurements in the field. During the later part of the time Mr. L. F. Penny, Head of the Geological Department, helped with the geological interpretation and joined the team in the field work. The team was always made up with other colleagues and occasional research students, and in particular Mr. A. L. Binns, whose regular attendance and help in providing some of the essential transport was always greatly appreciated.

The Mendip survey was carried out with members of the University of Bristol Spelaeological Society, who have always been willing to help when help was needed.

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Finally, I would again like to express my thanks to the Royal Society for a grant covering the cost of the instruments and some of the accessory equipment.

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(7) APPENDIX

The chief theoretical problems involved in geoelectric surveying are (i) the derivation and evaluation of the potential functions $\rho_s = f(V)$ or $f(R)_{Iconst}$ for different electrode configurations and for specific geological anomalies, and (ii) the solutions of the differential equations $d^2(\rho_s)/da^2 = 0$ and $d(\rho_s)/da = 0$, leading to relationships between the depths of anomalies and the values of *a* for points of inflection and for changes in gradients of the graphs of the potential function.

Ability to derive the potential equation for a given geological anomaly depends to some extent on the simplicity of the electrode configuration. Some authorities^{20–23} attack the problem by the direct solution of Laplace's equation involving Bessel functions, or deduce the potential distribution over the earth's surface as a real integral leading to more or less rapidly converging series. These fundamental methods are of considerable academic interest but of less practical value in the field than the solutions (sometimes only approximate) which arise by the use of Maxwell's method of electric images. Although the solutions of equations derived by this method also involve the summation of series, they have been found more convenient in practice and therefore have been adopted for the derivation in this Appendix of eqns. (2), (3) and (5).

(7.1) The Derivation of Eqn. (2)

Assuming that the potential field at a distance d from a point source of strength I in a homogeneous medium of resistivity ρ is $\rho I/4\pi d$, then for a half-space with a source on the boundary, this becomes $\rho I/2\pi d$ because the effective current strength is 2I, the strength of the original source plus that of its electric image in the boundary. This assumes the boundary is a perfect reflector, which will be the case if the resistivity of the medium above the boundary is infinite, as for air.

With a source C_1 and a sink C_2 , both on the boundary and separated by a distance 2*a*, the potential at a point P on the line joining the source to the sink and situated a distance (a - b)from the source and (a + b) from the sink, will be

$$V_P = \frac{\rho I}{2\pi} \left(\frac{1}{a-b} - \frac{1}{a+b} \right)$$

Consequently the difference in potential between two points P_1 and P_2 situated symmetrically between the points C_1 and C_2 will be

$$(V_{P1} - V_{P2}) = \frac{\rho I}{2\pi} \left[\left(\frac{1}{a-b} - \frac{1}{a+b} \right) - \left(\frac{1}{a+b} - \frac{1}{a-b} \right) \right] = \frac{\rho I}{\pi} \frac{2b}{a^2 - b^2}$$

On substituting α for the ratio b/a and solving for ρ we get

I

$$p = \frac{\pi(1-\alpha^2)}{2\alpha}a\frac{V}{I}$$
 or $\frac{\pi(1-\alpha^2)}{2\alpha}aR$

which is eqn. (2). Eqn. (1), for the Wenner technique, is the special case when $\alpha = 1/3$.

(7.2) The Derivation of Eqn. (3)

This equation gives the potential function for two parallel strata, the upper or superficial layer being of thickness or depth h and of resistivity ρ_1 , and the lower being of infinite thickness and of resistivity ρ_2 . Following the optical analogy, the reflection coefficient of the earth/air boundary will be $(\infty - \rho_1)/(\infty + \rho_1)$, or unity. This assumes the resistivity of the air above the boundary is infinite. The reflection coefficient k for the boundary between the parallel layers will be $(\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ for 'light' passing from the upper to the lower medium. Electric images of the source 2I will be of intensity 2kI at distances 2h above and below the earth/air boundary. These primary images will produce secondary images of strength $2k^2I$ at distances 4h above and below the surface boundary, and so on. (See Fig. 14.)



Fig. 14.—Electric image system for two parallel strata.

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These additional sources will all contribute to the potential at a point P_1 situated on the surface at a distance d from the source. Thus the potential at P_1 will be

$$V_{P1} = \frac{\rho I}{2\pi} \left\{ \frac{1}{d} + 2 \sum_{n=1}^{n=\infty} k^n [d^2 + (2nh)^2]^{-1/2} \right\}$$

There will be a similar contribution but of opposite sign to the potential at P₁ if a sink is also situated on the boundary. Now let it be supposed that a source 2*I* at C₁, and a sink -2I at C₂ are placed on opposite sides of a station S, so that $C_1S = SC_2 = a$. Also let two potential points P₁ and P₂ be similarly placed but at distances b from the station, all five points being in a straight line. Also let $b/a = \alpha$ and be less than unity. Then, to determine the difference of potential between P₁ and P₂, it will be necessary to subtract the total field at P₂ (due to both source and sink and their respective images) from the total field at P₁. There will be four terms similar to that given above for V_{P1} , but with d replaced by either (a - b) or (a + b). The resulting algebraic summation is

$$(V_{P1} - V_{P2}) = \frac{\rho I}{\pi a} \frac{2\alpha}{(1 - \alpha^2)} \\ \left[1 + \frac{\alpha}{(1 - \alpha^2)} \sum_{n=1}^{n=\infty} k^n \left\{ [(1 - \alpha)^2 + (2n\beta)^2]^{-1/2} - [(1 + \alpha)^2 + (2n\beta)^2]^{-1/2} \right\} \right]$$

in which $\beta = h/a$. Eqn. (3) now follows directly by substituting from eqn. (2) in the first part of this expression.

Each one of the family of curves represented by eqn. (3) has one point of inflection which will have some ordinate value a_0 , say. If the relationship between a_0 and h could be determined, the thickness h of the upper stratum could be calculated from a knowledge of a_0 taken from the graph. The required relationship can be obtained by solving the equation

$$d^2(\rho_s)/da^2 = 0$$

This is not easy to do, but graphical solutions have been obtained (see Reference 4, Fig. 2) for different values of α and k. A very approximate solution useful for rough field work is $h = 0.5a_0$, but as the ratio of h/a_0 depends on the values of both α and k, any more exact relationship must be read off the (α, β) graph for the correct value of k. For example, with k = -0.5 and $\alpha = 0.8$, the value of β is 0.47, i.e. $h = 0.47a_0$. With k = +0.5 and the same value of α , β is equal to about 0.35, or $h = 0.35a_0$. If $\alpha = 0.9$ with k = -0.5, the ratio h/a_0 drops from 0.47 to 0.33. In general, the higher the values of α or k, the smaller will be the ratio h/a_0 .

DISCUSSION BEFORE THE MEASUREMENT AND CONTROL SECTION, 3RD FEBRUARY, 1959

Professor S. K. Runcorn: The surveys most frequently made by geophysicists are magnetic and gravitational. These are mainly of use to the oil industry, where it is very important to obtain information of the structure of strata at fair depths. On the other hand, the geoelectric survey is not usually very effective in determining the structure at great depths, although, as the author points out, it is extremely efficient in detecting, for instance, the depth of bedrock. Such surveys thus have considerable application to engineering geology.

It is worth noting that there is a fundamental difficulty in inferring the structure of the rocks below the earth's surface by geophysical survey. This is well illustrated by considering the gravitational or magnetic cases, in which the intensity of the earth's gravitational field or the strength of one of the components

(7.3) The Derivation of Eqn. (5)

The derivation of the potential function for a sphere of radius r buried to a depth h in a medium of resistivity ρ is most easily carried out by replacing the sphere with an electric doublet Q and its image Q' situated a distance h below and above the boundary respectively. The two half-spaces are replaced by a homogeneous medium of resistivity ρ and of infinite extent. The current source and sink must be doubled in intensity as before. This is a common hydrodynamic problem, and the potential at any point P is given by

$$V_P = \frac{\rho I}{2\pi} \left[\frac{1}{d_1} - \frac{1}{d_2} + \frac{a d_0 r^3}{(a^2 + h^2)^{3/2}} \left(\frac{1}{d_3^3} + \frac{1}{d_4^3} \right) \right]$$

where the distances d and a are as shown in Fig. 15.



Fig. 15.—Electric doublet and image system for a sphere buried in a homogeneous medium.

To obtain the potential difference between two points P_1 and P_2 situated distances +b and -b from the origin on the line joining the source and sink, d_0 becomes +b or -b and the other values of d are the respective radius vectors of P_1 and P_2 . On making the appropriate substitutions in the above value

for V_P we get

$$(V_{P1} - V_{P2}) = \frac{2\alpha\rho I}{\pi(1 - \alpha^2)a} \left\{ 1 + \frac{\beta^3(1 - \alpha^2)(r/h)^3}{\alpha^3(1 + \beta^2)^{3/2} [1 + (\beta/\alpha)^2]^{3/2}} \right\}$$

where $\alpha = b/a$ and $\beta = h/a$ as before. Eqn. (5) now follows directly by substituting from eqn. (2) in the first part of this expression. It may be noted that, in the absence of the sphere, i.e. when either $r \rightarrow 0$ or $h \rightarrow \infty$, the above equation reduces directly to eqn. (2) for a homogeneous medium.

of the earth's magnetic field is measured over the surface. One then has to solve an inverse potential problem, i.e. to determine what spatial variations of attracting matter can account for the observed field variation at the surface.

It is important to realize that there is an essential ambiguity. For example, suppose that a gravitational survey is made over a surface below which is a substratum of rather dense rock which has been faulted. The disturbance will cause an anomaly in the earth's gravitational field, and the variation observed as one moves across the surface will be fairly sharp. If this discontinuity is much deeper, the field variation at the surface is much smoother but this same variation could equally well result from a more gradual change of density laterally at a shallower depth. Thus it is impossible to infer from the surface survey the depth of the